Q1a

a)

$\frac{\pi}{2} = 1.57$

ASYMPTOTE (DISCONTINUITY) AT $\frac{\pi}{2}$

SO

GRAPH IS NOT CONTINUOUS THROUGH THE INTERVAL $1.5 < x < 1.6$

Q1b

b)

$f(0) + 3 = 0$

TRANSLATION UP BY 3

$0 \tan(\pi - 0) - 3 + 3 = 0$

$0 - \cancel{3} + \cancel{3} = 0$

$f(0) + 3 = 0$ SO $x = 0$ IS A SOLUTION TO $f(x) + 3 = 0$

Q1C

c)

$\boxed{\text{F(x)+3 IS A TRANSLATION UP BY 3 EITHER SIDE OF } x=0 \text{ WILL GIVE NEGATIVE ANSWER AS } f(x)+3 \text{ TOUCHES } x \text{ AXIS RATHER THAN CROSSING IT (TURNING POINT)}}$

Q2a

a) $\frac{1}{e^x} - x + 1 = 0$

$\frac{1}{a^x} = a^{-x}$

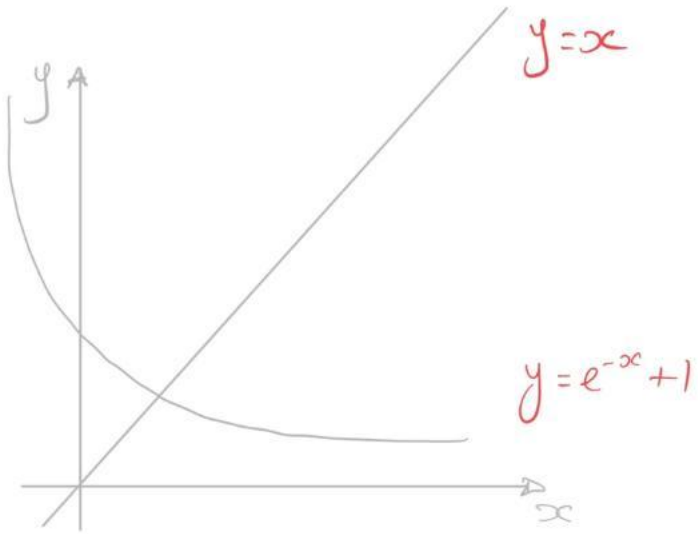
$e^{-x} - x + 1 = 0$

$e^{-x} + 1 = x$

$\boxed{x = e^{-x} + 1}$

Q2b

b)



TRANSFORMATION

$y = e^{-x} + 1$

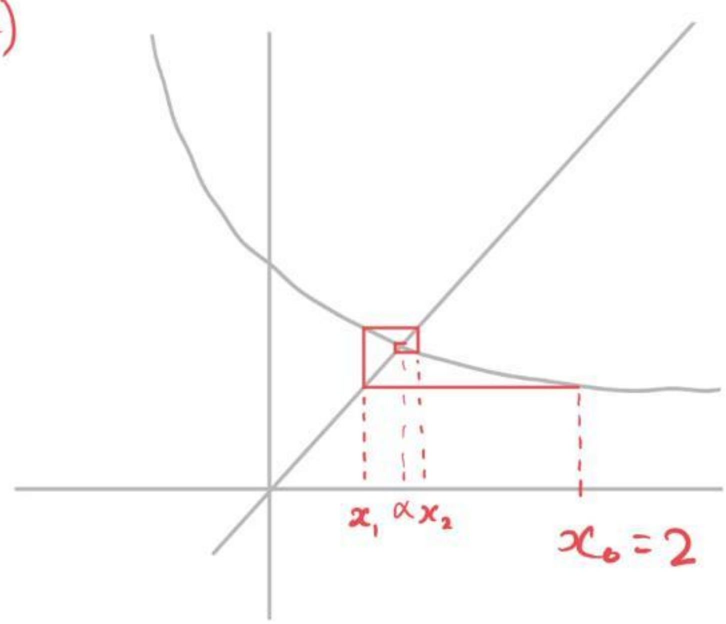
e^{x} e^{-x}

REFLECTION IN Y AXIS

$+1$ TRANSLATE UP 1

Q2c

c)



Q2d

d)

i) $x_0 = 2$

$x_1 = 1.135335...$

$x_2 = 1.321314...$

$x_3 = 1.266784...$

$\boxed{x_1 = 1.14,\ x_2 = 1.32,\ x_3 = 1.27}$

ii) $x_4 = 1.2817...$ 1.28

$x_5 = 1.2775...$ 1.28

$\boxed{\text{5 ITERATIONS}}$

Q2E

e) $x = 1.28$ (2dp) BOUNDS

LB UB

1.275 1.285

$\boxed{\begin{array}{l} p = 1.275 \\ q = 1.285 \end{array}}$

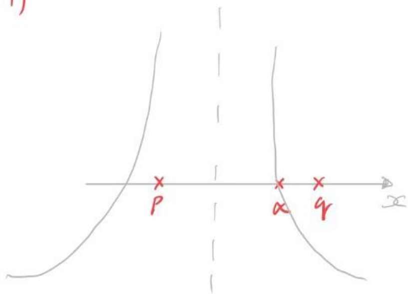
Q3

Sketch three separate graphs with values of $x = p$ and $x = q$, to show how the sign change rule would fail to find a root α in the interval $(p\,,q)$ for the following reasons:

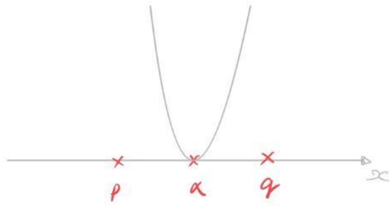
(i) Sign change rule indicates a root but there isn't one due to a discontinuity in the graph.
(ii) Sign change rule indicates no root but there is a root at a turning point.
(iii) Sign change rule indicates no root but there are in fact two roots in the interval $(p\,,q)$.

On each diagram, clearly labelled p, q and the root α.

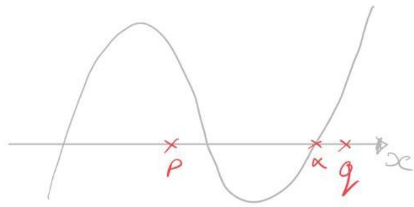
i) DISCONTINUITY (ASYMPTOTE)



ii) TURNING POINT TOUCHES AXIS

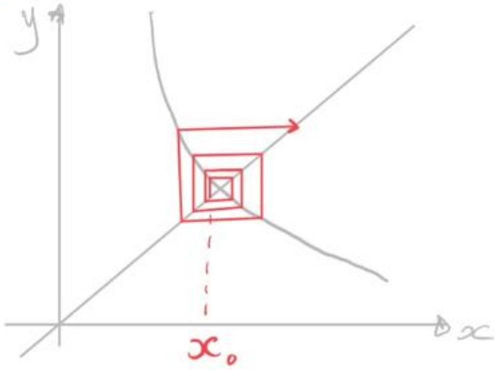


iii) MULTIPLE ROOTS

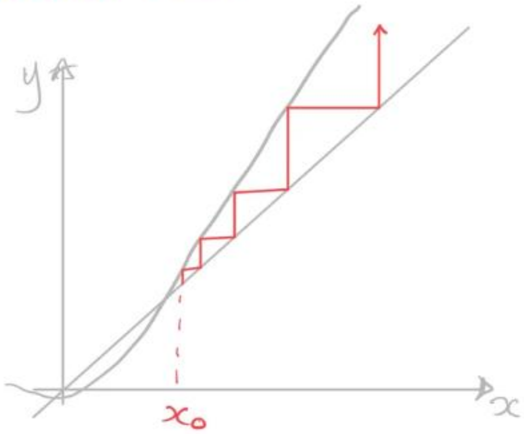


Q4

COBWEB



STAIRCASE OR LADDER



Q5A

a) $x = 1 + 2\ln(x^2)$

$1 = 1 + \cancel{2\ln(1^2)}$

INTEGER VALUE LESS THAN 5

$\ln(1) = 0$

$\boxed{S = (1, 1)}$

Q5b

b) i) STAIRCASE / LADDER DIAGRAM WITH ANY APPROPRIATE x_0

ii) $x_{n+1} = 1 + 2\ln(x_n^2)$

$x_0 = 10$

$x_1 = 10.21034\ldots$

$x_2 = 10.29360\ldots$

$x_3 = 10.32690\ldots$

$x_4 = 10.338695\ldots$

$x_5 = 10.34374\ldots$

$x_6 = 10.345462\ldots$

$x_7 = 10.3461919\ldots$

$x_8 = 10.346474\ldots$

$\boxed{p = 10.346 \text{ (5sf)}}$

Q5c

c) $S=(1,1)$ $P(10.346, 10.346)$

PYTHAGORAS

$\sqrt{2} \times 9.346$

$= 13.217\ldots$

$\boxed{13.2 \text{ MILES OR km}}$

Q6

LB $x_{n+1} = \ln(3x_n+4) - 0.25x_n^2$

$x_0 = 1.5$

$x_1 = 1.57756\ldots$

$x_2 = 1.5448\ldots$

$\vdots$ $x = 1.6$ (2sf)

UB $x_{n+1} = \ln(3x_n+4) - 0.25x_n^2 + 1$

$x_0 = 2.1$

$x_1 = 2.229643\ldots$

$x_2 = 2.1263\ldots$

$\vdots$ $x = 2.2$ (2sf)

PYTHAGORAS TO FIND DISTANCE FROM $(0,0)$

LB $\sqrt{2 \times 1.6^2} = \sqrt{2} \times 1.6 = 2.262\ldots$

UB $\sqrt{2 \times 2.2^2} = \sqrt{2} \times 2.2 = 3.11\ldots$

$\boxed{LB = 2.3 \text{ (2sf)} \quad UB = 3.1 \text{ (2sf)}}$

Q7a

a)

$t_0 = 10$

$t_1 = 9.93962\ldots$

$t_2 = 9.963849\ldots$

$t_3 = 9.95411\ldots$

$t_4 = 9.95802\ldots$

$t_5 = 9.95645\ldots$

$\boxed{t = 9.96 \text{ DAYS } (3\text{sf})}$

Q7b

b) i) REARRANGE $f(t)$ TO FIND A

$$Ae^{-0.25t} = 0.1t$$

$$e^{-0.25t} = \frac{t}{10A} \qquad 0.1t = \frac{t}{10}$$

$$-0.25t = \ln\left(\frac{t}{10A}\right)$$

$$-\ln(x) = \ln(x)^{-1} \qquad 0.25t = \ln\left(\frac{10A}{t}\right)$$

$$x^{-1} = \frac{1}{x}$$

$$t = 4\ln\left(\frac{10A}{t}\right)$$

$$10A = 120 \qquad \boxed{A = 12}$$

ii) $v(T) = 12Te^{-T} - 0.1T$

$$T = \ln\left(\frac{120\cancel{T}}{\cancel{T}}\right) = \ln(120) = 4.7874\ldots$$

$$\boxed{T = 4.79 \text{ SECONDS}}$$

Q7c

c) WITHOUT = 9.96 DAYS

WITH = 4.79 SECONDS

$9.96 \times 24 \times 60 \times 60$ CONVERT TO SAME UNITS

$= 860544$ SECONDS

$$\frac{860544}{4.79} = 179654.2797$$

$\boxed{180{,}000 \text{ TIMES QUICKER}}$